A model for neural representation of intervals of time

H. Okamoto\textsuperscript{a,b,*}, T. Fukai\textsuperscript{b,c}

\textsuperscript{a}Corporate Research Labs., Fuji Xerox Co. Ltd., 430 Sakai, Nakai-machi, Ashigarakami-gun, Kanagawa 259-0157, Japan
\textsuperscript{b}CREST of JST (Japan Science and Technology Corporation), Japan
\textsuperscript{c}Department of Information-Communication Engineering, Tamagawa University, Tamagawagakuen 6-1-1, Machida, Tokyo 194-8610, Japan

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Abstract

We put forward a model for neural representation of intervals of time. The model is comprised of ordinary recurrent neural networks. Assumptions specific to our model are the following two: membrane potential of each neuron is bistable; each neuron receives random noise input in addition to the recurrent input. Results of computer simulation show that the network activity triggered at an initial time continues for prolonged duration followed by an abrupt self-termination. This time course seems quite suitable for representation of intervals of time. Weber’s law, a hallmark of human and animal interval timing, is also reproduced.

Keywords: Interval timing; Recurrent network; Bistability; Stochastic dynamics; Weber’s law

1. Introduction

Capabilities of humans and animals to detect, store and recall intervals of time have been demonstrated by a number of cognitive and behavioural studies [5]. Timing an interval between conditioned stimulus and delayed delivery of reward is one of the most frequently used experimental paradigms for such studies [10]. However, despite that it is thus obvious at cognitive and behavioural levels that humans and animals have ability of interval timing, little is known about its neural mechanisms [2,5,6,8].

*Correspondence address: Corporate Research Labs., Fuji Xerox Co. Ltd., 430 Sakai, Nakai-machi, Ashigarakami-gun, Kanagawa 259-0157, Japan. Tel.: +81-465-80-2015; fax: +81-465-81-8961.
E-mail address: hiroshi.okamoto@fujixerox.co.jp (H. Okamoto).

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the present study, a model for possible mechanisms of neural representation of intervals of time is examined on the basis of physiologically probable hypotheses.

Psychophysical studies of human interval timing have established that the standard deviation $\sigma$ of response time is a linear function of the mean $\mu$, that is, the Weber ratio $\sigma/\mu$ is constant with $\mu$ (Weber’s law) [5]. This linear relationship is also observed in animal interval timing [5]. Thus, Weber’s law is a hallmark of human and animal interval timing. Although a variety of hypothetical models for neural mechanisms of interval timing may be considered, plausibility of each can be tested by examining whether it can reproduce Weber’s law. We will see that the model examined in the present study passes this test.

2. Model

Our model is extremely simple and general. We consider ordinary recurrent networks of $N$ neurons (Fig. 1). Only two assumptions that are specific to our model are added. The first one is that: If sufficient recurrent input is provided, membrane potential of each neuron is bistable. This means that the firing rate of a neuron is high/low if this neuron is at the up/down state. It has been proposed in recent theoretical studies that voltage dependence of NMDA conductance coupled with GABAergic conductance can lead to membrane potential bistability [3,7], which supports physiological probability of the above assumption.

The second assumption is that: Each neuron receives random noise input in addition to the recurrent input. A brain region responsible for interval timing, presumably located in the prefrontal cortex [4], should be regarded not as a closed system but as an open system interacting with a lot of other brain regions. Introducing random noise is the most general way to approximate interactions between an open system and an environment.

Rigorously, the time course of the activity of each neuron should be described by the Hodgkin–Huxley equation. In the present study, however, we use stochastic dynamics of a two-spin Ising system instead of the Hodgkin–Huxley equation. This substantially simplifies nonessential aspects of the model while keeping its essential features, i.e., bistability and randomness. A two-spin Ising system for the $n$th neuron is

![Fig. 1. Recurrent neural networks comprising our model.](image-url)
defined by the Hamiltonian
\[ H^{(n)} = -ws_1^{(n)}s_2^{(n)} + (\theta - I)(s_1^{(n)} + s_2^{(n)}). \]  
(1)

Here, \( s_1^{(n)}(i = 1, 2) \) is a spin variable taking 1 or 0. \( I \) represents the recurrent input, defined by
\[ I = GN_{up}/N, \]  
(2)

where \( N_{up} \) is the number of neurons at the up state, and \( G \) is the synaptic strength which, for simplicity, is assumed equal everywhere in the networks. If \( I \) satisfies
\[ \theta - w < I < \theta \]  
(3)

the system is bistable; that is, there are two stable states; \( (s_1^{(n)}, s_2^{(n)}) = (1, 1) \) and \( (s_1^{(n)}, s_2^{(n)}) = (0, 0) \). The former and later are assigned to the up and down states of the \( n \)th neuron, respectively. If \( I \) becomes smaller to satisfy
\[ I < \theta - w \]  
(4)

there is only one stable state; \( (s_1^{(n)}, s_2^{(n)}) = (0, 0) \).

The following algorithm gives stochastic dynamics of the Ising system:
\[ s_1^{(n)}(t + \Delta t) = 1 \] with probability \( p_1^{(n)}(t) = 1/(1 + \exp(-\beta H^{(n)})) \)  
(5a)

\[ s_1^{(n)}(t + \Delta t) = 0 \] with probability \( 1 - p_1^{(n)}(t) \)  
(5b)

This is just the same stochastic algorithm as that used for Boltzmann machine [1].

3. Results and discussion

Define the network activity \( P \) by the ratio of the number of up state neurons to the total number of neurons:
\[ P = N_{up}/N \]  
(6)

The time courses of \( P \) obtained by several trials of computer simulation for the same parameter set but with different random numbers are superimposed in Fig. 2. The network activity triggered at an initial time is sustained for prolonged duration, followed by an abrupt self-termination. The duration scatters because of the stochastic nature of the model. This time course seems quite suitable for neural representation of intervals of time, as suggested by electrophysiological recording from prefrontal and cingulate cortex during timing behaviour [9].

One can easily see that, as the parameter \( w/\theta \) increases, the up state becomes more stable while the down state becomes less stable. Hence, \( w/\theta \) is considered to represent “cell excitability”. For different \( w/\theta \), different \( \mu \) and \( \sigma \) are obtained (data not shown). However, the Weber ratio \( (\sigma/\mu) \) appears to remain constant (Fig. 3). This confirms plausibility of our model.
Fig. 2. Time course of the network activity $P$. Each curve represents the results of numerical simulation based on the stochastic algorithm (described in the text) for the same parameter values but with different random numbers. $N = 500; \beta \theta = 60.0; G/\theta = 0.8; w/\theta = 0.33; \Delta t = 0.01$ s.

Fig. 3. The Weber ratio ($\sigma/\mu$) calculated for different $w/\theta$. The values for the other parameters are the same with Fig. 2.

Emergence of the characteristic time course of $P$ shown in Fig. 2 can be accounted for as below. At $t = 0$, all the neurons are set at the up state. In this stage, $I$ is strong enough to make each neuron bistable. If the dynamics were totally deterministic, each neuron would eternally remain at the up state. In our model, however, there is a probability for each neuron to escape from the up to down states. Therefore,
gradually decreases as time progresses. Accordingly, \( I \) decreases as well (see Eq. (2)). When \( I \) crosses the value \( \theta = \omega \), the potential form for each neuron changes from bistability to monostability (see Eqs. (3) and (4)). Then, the remaining up-state neurons all together switch to the down state. This is just analogue to first-order phase transition in statistical physics.

Escape of neurons from the up to down states with the help of random noise progresses asynchronously and very slowly. In contrast, switching of neurons from the up-to-down states at the phase transition point happens almost synchronously and very quickly. These explain the prolongation and the abrupt self-termination of the network activity.

References


Hiroshi Okamoto specialized in Theoretical Physics and obtained Ph.D. at Waseda University in 1991. Since then, he has been working as a researcher at Corporate Research Labs, Fuji Xerox Co. Ltd. His research theme is to create “knowledge technology” inspired by brain science. He is interested in theoretical studies of neural information processing both at cellular and at network levels. He is a member of a research project supported by CREST of JST (Japan Science and Technology Corporation) and directed by the co-author Dr. Fukai.

Tomoki Fukai received the Ph.D. degree in Physics from Waseda University. He worked on theories of Particle Physics. His current research interests focus on computational theories of neural systems, both artificial and biological, and physics of nonlinear systems. At present, he is an Associate Professor of Department of Information-Communication Engineering, Tamagawa University, Tokyo, Japan. He is also serving as a leader for a research project supported by CREST of JST.